End-Effects in Quasi-TEM Transmission Lines

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Abstract— **Magnetostatic analysis of a finite-length two-wire transmission line yields simple closed-form expressions for inductive end-fringing and interaction between ends. A further argument relates the results to capacitive end effects. Application to microstrip-like lines, twin-strip line and coplanar waveguide is outlined. It is demonstrated by explanation and comparison with the literature that these effects are the dominant discontinuity elements in short lengths of line, vias, resonators, bends, and other basic microwave configurations.**

I. Introduction

At THE PRESENT time, most microwave circuit design activity is carried out with powerful programs based on circuit theory because of their flexibility of use, their ability to handle complicated and extensive circuit configurations, and the rapidity with which complete circuit analyses can be effected. Such circuit analysis programs are themselves based in part on simple mathematical descriptions (models) of a number of basic microwave transmission-line elements and discontinuities. The most brief and convenient models are expressed as closed-form algebraic relations. These take little program or storage space, compute very rapidly, and if derived from theory, are usually applicable over very wide parameter ranges.

In this paper, the formula for the external inductance of a finite length of abruptly ended, two-wire line [1] is derived. Analysis shows that the formula can be divided into three simpler closed-form expressions that describe inductance per unit length, end-effect inductance and interaction inductance between the ends. A related argument gives similar expressions for capacitive end-effects. The end-effect elements can be considered to be localized at the ends of the line and lumped.

A reviewer has brought to the author's attention the work of R. W. P. King and K. Tomiyasu [2], who previously developed a more general theoretical approach, based on electromagnetic potentials, for the analysis of variously terminated two-wire lines. A more extensive presentation of the work was published subsequently by King [3]. These references have corroborated the expressions for inductive and capacitive end-effects of this paper, and also have provided a sound theoretical basis for the derivation of capacitive end-effects. This paper differs from [2] and [3] not only in the theoretical approach, but also in introducing the concept of and expressions for end-interaction inductance. In addition, and of importance for practical application, this paper shows that the models for the two-wire discontinuity elements can be related to widely-used, modern planar transmission lines. Explanation and comparison with published discontinuity data is provided for certain representative microwave structures. Some applications of the model are: abruptly terminated microstrip lines, resonators, vias, ground-wraps, open-ended stubs, right-angle bends, tee-junctions, hairpin bends and dogleg bends.

In summary, the innovations of this paper are:

1. to analyze the formula for the inductance of a finite two-wire line in terms of transmission line inductance, end-effect inductance and end-interaction inductance, realizing simple closed-form relations for each;
2. to show that there exists a simple relation between the above inductive elements and corresponding capacitive end-elements;
3. to show that these end-elements can be applied to unbalanced microstrip-like lines, and that they characterize the dominant discontinuity effects in many practical microwave transmission-line configurations.

II. The Inductive Model

The external inductance of a pair of parallel wires of finite length is derived from first principles in the Appendix. The derivation finds the contribution of one wire of the pair to the total external inductance of a rectangular loop of wire, as shown in Fig. 1(a). The horizontal round conductors are the wires of interest. The vertical end-wires run at right angles to the conductors of interest, and thus do not couple, except to each other. They can be considered filamentary, and will be dropped from further consideration in this work. The notation used in Fig. 1(a) and the equations of this paper is presented next.

A is the distance between hypothetical filamentary currents generating magnetic fields identical to those of a very long wire pair, external to the wire surface [4].

B is the center-to-center spacing of the round wires. D is the diameter of the wire.

$$A = (B^2 - D^2)^{1/2} \tag{1}$$

$G = B - D$, the width of the space between the wires. (2)

S is the length of the wires.

$$P = (A - G)/2 \qquad Q = (A + G)/2 \tag{3}$$

P and Q are convenient parameters used as integration limits.

$$R = D/2$$
$$\omega = \text{radian frequency}$$
$$\mu = 0.4\pi \text{ nH/mm}$$
$$\eta_0 = 376.73 \text{ Ohms/square}$$

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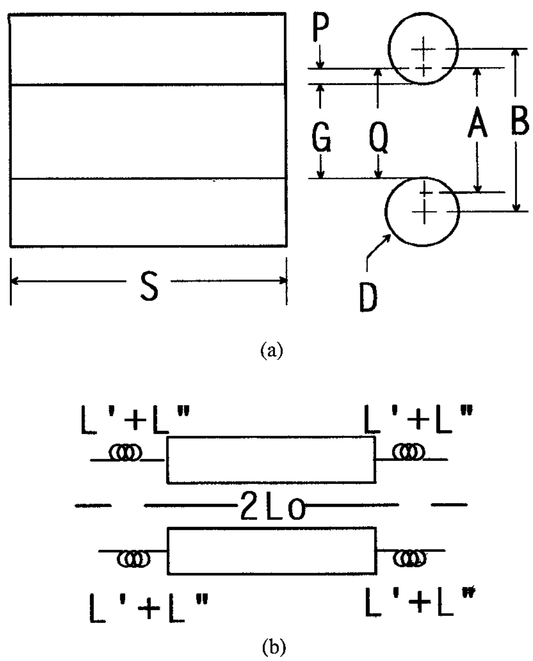


Fig. 1. (a) Sketch of finite-length parallel conductors. (b) Equivalent circuit of finite-length parallel conductors.

The well-known formula for the inductance, L_1, of a single isolated wire falls out of the analysis. It is derived in the Appendix and given here for subsequent use.

$$L_1 = (\mu/2\pi)S[R/S - (1 + (R/S)^2)^{1/2} + \log((1 + (1 + (R/S)^2)^{1/2})/(R/S))] \quad (4)$$

The inductive model for the finite-length two-wire line is shown in Fig. 1(b). It is comprised of three types of inductive elements: L_o, L' and L''. Observe that these are defined with respect to a single conductor and centered ground plane (or electric wall), for simple application to unbalanced lines, such as microstrip. L_o/S is the familiar inductance per unit length associated with infinite transmission lines:

$$L_o = (\mu/2\pi)S\ln(Q/P) = (\mu/2\pi)S\cosh^{-1}(B/D). \quad (5)$$

This expression is consistent with the literature [4, Table 9.01], and holds for all wire diameters and spacings L' is a negative inductance that is independent of the length of the line, and therefore, can be associated with the ends of the line. It is negative to compensate for the reduction of magnetic field near the ends of the line.

$$L' = -(\mu/4\pi)G = -(\mu/4\pi)Q(1 - P/Q) \quad (6)$$

L'' is a positive inductance representing coupling of currents at the opposite ends of the line. Its value just cancels the value of L' at zero line length, and rapidly diminishes as line length increases. Thus, it is significant only for very short lines. For this reason, L'' can be considered lumped at the ends of the line.

$$L = (\mu/4\pi)S \left\{ \cdot \ln\left[\frac{1 + (1 + (P/S)^2)^{1/2}}{1 + (1 + (Q/S)^2)^{1/2}}\right] + (1 + (Q/S)^2)^{1/2} - (1 + (P/S)^2)^{1/2} \right\} \quad (7)$$

Within the limits of his small-diameter approximation, King's expression [3, p. 121] for end-inductance agrees with (6). King does not determine interaction inductance, L''.

III. Relations to Other Transmission Lines

The results of this work can be applied to other transmission lines having physical similarity to two-wire line; of particular interest, open quasi-TEM lines. Thus, abrupt discontinuities on twin-strip and open (or highly unbalanced) microstrip-like lines would be expected to behave much as predicted by this two-wire line model.

The following relations have been found to relate a specific microstrip transmission line to the two-wire discontinuity models:

1. The effective dielectric constant of the model is made the same as that of the microstrip.

2. The parameter Q is set to twice the microstrip conductor-to-ground-plane spacing, H. Parameter Q is the distance from the inner side of one conductor to the location of the equivalent filamentary current of the other:

$$Q = 2H \quad (8)$$

3. The characteristic impedance of the model is set equal to twice that of the microstrip by imposing the appropriate diameter-to-spacing ratio, D/B, or parameter ratio, P/Q. It is well-known that characteristic impedance of a TEM line is directly related to inductance per unit length, as shown in (9) in the notation of this paper, with Z_{oa} the characteristic impedance of an air-filled, unbalanced strip conductor line, such as microstrip:

$$Z_{oa}/\eta_0 = (L_0/S)/\mu \quad (9)$$

Thus, from (5), the value of P is

$$P = \exp(-2\pi Z_{oa}/\eta_0) \quad (10)$$

Parameters Q and P are now known in terms of microstrip substrate thickness and characteristic impedance, and so the microstrip end-effect inductance and interaction inductance can be found. The important negative inductance, L', (6), becomes:

$$L'/H = -0.2(1 - \exp(-Z_{oa}/60))\,\text{nH/mm}. \quad (11)$$

Equation (11) should hold for all values of microstrip characteristic impedance, Z_{oa}, to the extent that (8), $Q = 2H$, is accurate. Comparison with the literature in a subsequent section shows that it is sufficiently accurate for practical use for all W/H up to at least 10, where W is conductor width. An approximate expression for L' that holds for all W/H up to about 2 or 3 (about 3% low at $W/H = 2.0$) is:

$$L'/H = -0.2(1 - (W/H)/8)\,\text{nH/mm}. \quad (12)$$

If interaction inductance, L'', is needed, it is best solved for in terms of P and Q, as given in (7), using (8) and (10). In typical cases, L'' has been found to be less than 10% of $-L'$ for line lengths greater than about 5 substrate thicknesses.

IV. The Capacitive Model

For the terminated uniform two-wire line, King [3, p. 68] derives a generalized propagation constant; that is, propagation constant as a function of position along the line relative to the termination. For neither magnetic nor electric coupling between line and termination, he shows that even as the termination is approached, the propagation constant does not change from its fixed, long line value. The following development is based on this observation; lossless line is assumed.

It is well known that inductance and capacitance per unit length, L and C, of a TEM line are related to its propagation velocity, v, by:

$$LC = 1/v^2 \tag{13}$$

For a short length, S, of abruptly terminated TEM line, such as shown in Fig. 1(a), let L_t be the total inductance and C_t be the total capacitance. The line is physically uniform over its length, and so (13) would be expected to hold:

$$L_tC_t = (S/v)^2 \tag{14}$$

As shown in the preceding section, for a short unbalanced line such as microstrip the total inductance can be expressed in terms of an inductance per unit length, L_o/S, and an inductance, $2(L'+L'')$, representing end effects. These will be designated now as L and L_e, respectively. Similar capacitive terms, C and C_e, must also exist. Therefore, (14) can be written as:

$$(SL + L_e)(SC + C_e) = (S/v)^2 \tag{15}$$

Expanding (15) and solving for end-effect element C_e yields:

$$C_\epsilon = -(L_eC/L)/(1 + L_e/SL) \tag{16}$$

Returning to the standard notation of this paper;

$$C' + C'' = \frac{-[(L' + L'')/Z_o^2]}{[1 + 2(L' + L'')/L_o]} \tag{17}$$

C' and C'' can be separated by arbitrarily requiring that C' be independent of line-length and observing that C'' should approach zero as the line becomes very

$$C' = -L'/Z_o^2 \tag{18}$$

Within the limits of his small-diameter wire approximation, King's expression [3, p. 367] for end-capacitance is consistent with C' of (18).

In Fig. 7 of Benedek and Silvester [5], total air-dielectric microstrip capacitance is normalized against the parallel-plate capacitance of the strip; the ordinate is given as C_tH/ϵ_oWS. Introducing (14) to define C_t in terms of L_t, this quantity is found to be equivalent to $\mu S/(W/H)L_t$. The latter form was used to calculate the curves of Fig. 2 of this paper, which shows computations of C_t based on (15) and some results from [5], using the normalized form. This demonstrates that the total capacitance of a short, abruptly terminated line can be predicted from its inductance for all aspect ratios.

Application of the capacitive end-effect formula, (18) to microstrip introduces a new problem. The analysis assumed homogeneous dielectric, an assumption which for most purposes can be handled by the concept of effective dielectric constant. For microstrip the effective dielectric constant, ϵ_e, experienced by the field determing Z_o and propagation velocity includes the effect of parallel-plate capacitance under the strip, while the end-effect effective dielectric constant does not. The end-effect effective dielectric constant, now denoted ϵ_e', will be made explicit as follows: (17) and (18) hold for any homogeneous dielectric, including vacuum; for other substrate dielectrics, C' and C'' are found from the solutions to those equations assuming vacuum dielectric, and multiplying by the end-effect effective dielectric constant, ϵ_e'. Thus, (18) becomes:

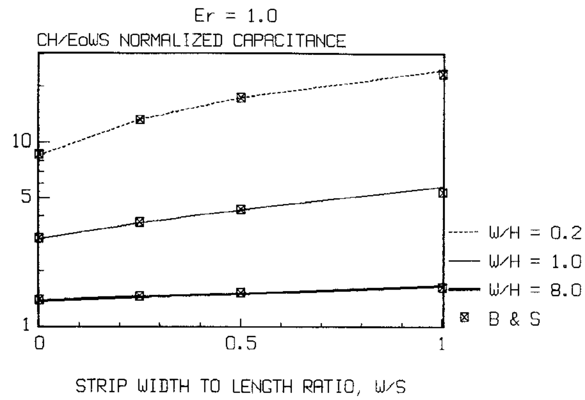


Fig. 2. Predicted total capacitance for finite-length microstrip.

$$C' = -\epsilon_e'L'/(Z_o^2\epsilon_e) \tag{19}$$

The quantity, $Z_o^2\epsilon_e$, is independent of ϵ_e. The problem now is to determine ϵ_e'. A theoretical, but approximate approach (not given here) to end-effect dielectric constant predicts ϵ_e' as:

$$\epsilon_e' = 1 + A(\epsilon_r - 1) \tag{20}$$

where ϵ_r is the relative dielectric constant of the substrate, and

$$A = 2\log(2)\pi = 0.4413.$$

Silvester and Benedek [6] in their Fig. 5 show end-effect capacitance against W/H for six substrate dielectric constants from 1.0 to 51.0. Their curves were computed by electromagnetic analysis using numerical techniques. Their end-effect effective dielectric constants were found to be reasonable fits to (20), after some adjustment of parameter A. Equation (19) using (20) with $A = 0.387$ is plotted in Fig. 3 for $\epsilon_r = 1., 9.6$, and 51., and compared with values computed by Silvester and Benedek [6]. Other work on capacitive end-effect was done by Itoh, Mittra and Ward [7], using a different electromagnetic technique than [6]. Their results for $\epsilon_r = 9.6$ are also shown in Fig. 3. The agreement with these sources is considered satisfactory corroboration of the capacitive end-effect formulas for microstrip.

V. The Shunt Tee Junction

In a shunt tee-junction, one transmission line is terminated at right angles on another, continuous line. The following analysis considers the slightly more general case of an impedance

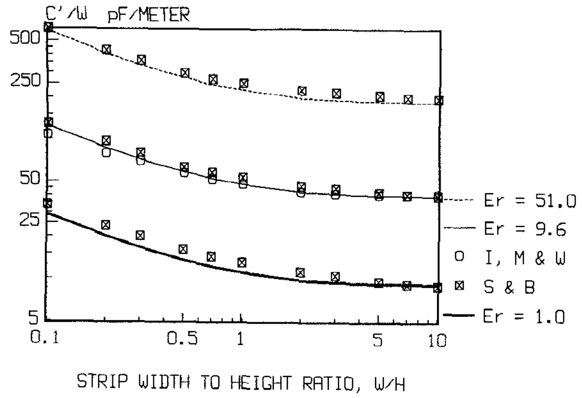


Fig. 3. Predicted end-capacitance for open ended microstrip.

connected, but not otherwise coupled, across a continuous transmission line.

Suppose a lumped impedance, Z, is connected across a transmission line, as shown in Fig. 4(a). Magnetic coupling between the line currents on either side of the junction is required to allow for different currents in the left and right halves of the connection. It can be accommodated by postulating inductors, La, with mutual inductance, M, as in Fig. 4(a).

Let the circuit be divided into left and right halves, and the halves be very widely separated; then M can be neglected, and the circuit for each half must be that of an abruptly terminated line. From the preceding arguments, La must be the negative element L', of (6), (11) or (12). Next, let the two halves of the circuit be brought together and rejoined as in Fig. 4(a). A completely equivalent tee-junction representation is shown in Fig. 4(b). For Z a very high impedance, the circuit of Fig. 4(b) must be that of a continuous transmission line. This requires the series elements, La-M, of Fig. 4(b) to be zero, and so the value of M is L'. But La and M are independent of Z. It can be concluded that the equivalent circuit of a transmission line across which an impedance is attached involves only the negative inductance, L', of the transmission line in series with the connected impedance.

The result can be summarized as follows:

"an otherwise uncoupled impedance connected aross a uniform transmission line always incurs a negative inductance equal to the abrupt-end-inductance, L', of the line."

As an example, the circuit for a tee-junction comprised of two transmission lines, A and B, is shown in Fig. 4(c). $L'(A)$ occurs in consequence of the above statement, and $L'(B)$ arises because line B is abruptly terminated at line A.

For rigor, it is necessary to state that the line conductors must be small compared to their spacing, and that impedance Z must be connected by a very small conductor. These restrictions can be relaxed, but the price is an uncertainty in terminal plane locations.

VI. Applications

This section describes some possible applications for the end-effect formulas derived in preceding sections.

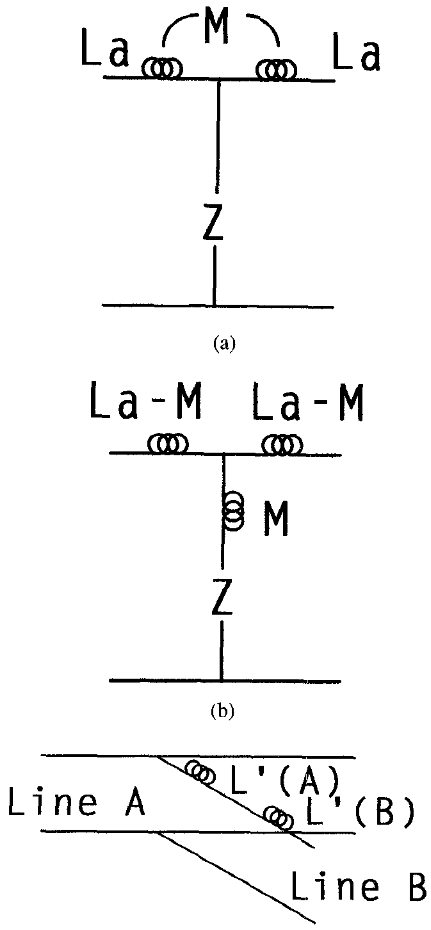


Fig. 4. (a)–(b) Circuit for transmission line with shunt impedance. (c) Circuit for transmission line with shunt junction.

Relations of End-Effect Elements to Electrical Length

It is sometimes advantageous to characterize an end-effect as an increment of line-length, dl. For conventional capacitance and inductance per unit length, C and L, dl is given by C'/C or L'/L, as appropriate. Characteristic impedance, Z_o, is usually a more convenient reference; in which case the above relations become:

$$dl = cC'(Z_o\epsilon_e^{1/2})/\epsilon_e$$
$$dl = cL'/(Z_o\epsilon_e^{1/2}) \tag{21}$$

where c is the speed of light in vacuum, and C' must include the end-effect effective dielectric constant. C' yields a positive value for dl, and L' yields a negative dl.

A measure of the interaction between the ends of an abruptly terminated line is L''/L', which ranges from 1.0 to 0.0 as the spacing between ends goes from 0.0 to infinity. For the microstrip of the preceding paragraph, this measure is 11% for a strip length of 5 substrate thicknesses, and 2% for a length of 28 substrate thicknesses.

The Abruptly Terminated Transmission Line

Consider a transmission line abruptly terminated by some connection to ground; perhaps a resistor. Both end-effect

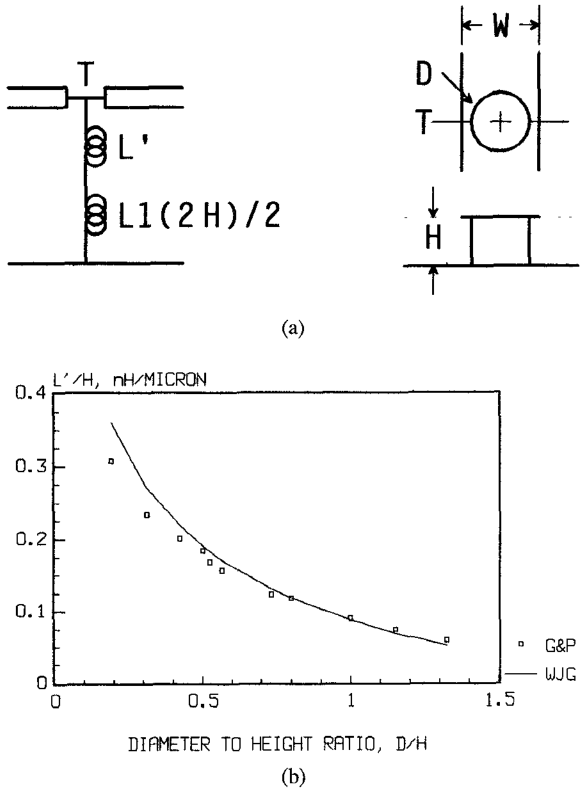


Fig. 5. (a) Via configuration and equivalent circuit. (b) Predicted via inductance.

elements, L' and C', may be included in the equivalent circuit, but a warning is in order: the inductive analysis depended on having no magnetic coupling between the line and any other circuit connected at the discontinuity. This required current flow at right angles; no similar isolating effect occurs with electric coupling. However, when both inductive and capacitive end-effects can be included, the approximate input impedance, Z_{in}, of the end-circuit terminated by impedance, Z_t, is:

$$Z_{\text{in}} = \frac{(Z_t + j\omega L')}{(1 + j\omega C' Z_t)} \tag{22}$$

This gives the expected values for open- and short-circuit terminations; for the "matched" termination, $Z_o = (L'/C'')^{1/2}$, the input impedance is $Z_{\text{in}} = Z_o - 2j\omega|L'|$. Thus, a perfect Z_o termination yields a mismatched line. For a 50-ohm line, a typical value for $2j\omega|L'|$ might be 10-ohms. In actual practice most terminations introduce a positive inductance that is greater than $|L'|$.

The Via

The via to be considered is a circular cylinder lying between a microstrip center-conductor and ground, as sketched in Fig. 5(a). Investigation of the via [8] illustrates concepts from the preceding sections:

First, the impedance presented by a via must include the negative inductive end-effect, L', whether the via terminates a line or shunts a continuous line, as argued in the tee-junction analysis presented above. The inductance of the via is that of an isolated wire given by (4). However, in order to allow for the reflection of the via current in the ground plane, it is necessary to solve (4) using twice the substrate thickness, and take half the resulting value for the via inductance. The total inductance of the via circuit is the via inductance plus the negative end-inductance, L', for the microstrip line, as shown in Fig. 5(a). Fig. 3. of [8] shows total via circuit inductance as numerically simulated by an electromagnetic analysis program. Some of the points from [8] are plotted in Fig. 5(b) and compared with the analysis of this paper.

Reference [8] does not give specific information on the widths of the microstrip lines used for the analysis. The writer assumed on the basis of Fig. 1 of [8], that the microstrip conductor widths were the same or close to the via diameter for each case. Also, the square top-pad mentioned in [8] was ignored in these computations. Despite these approximations, the agreement between the numerical simulation and the results of the method of this paper demonstrates the effectiveness of this method.

The Right Angle Bend

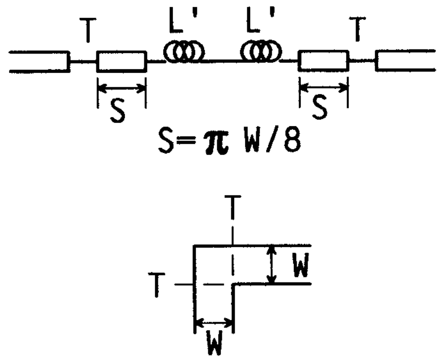
Both arms of the right angle bend, sketched in Fig. 6(a), have the same conductor width. The bend on microstrip incurs an L' for each arm, because the currents in the two arms are not coupled magnetically. In this theory, a single natural terminal plane exists at the point on the line at which the abrupt change in current direction occurs. For a bend of narrow conductors, the terminal plane can be taken to lie at the center of the diagonal of the bend. For wider conductors, the requirement of abrupt redirection of current is not met, and so there is not a clearly defined terminal plane.

A reasonable estimate for the location of an effective terminal plane for the bend is made by assuming that the wave on the line traverses the bend by following the quarter-circle that joins the centerlines of the two arms. The diameter of the circle is W, and so the length of the line going around the bend is $\pi W/4$. Thus, an effective terminal plane can be taken as lying a distance $\pi W/8$ from the inner corner for each arm. This is shown in the equivalent circuit of Fig. 6(a).

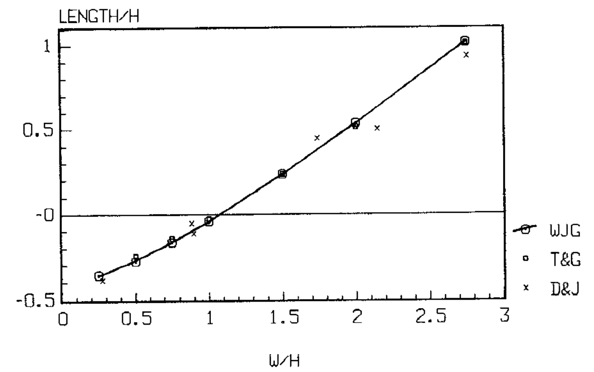
The usefulness of this approach is verified by comparison with other theory [9] and with measurements [10]. Figure 6 of [10] shows the effective electrical length, 2δ, designated S_e in this paper, between conventional inner-corner terminal planes (T of Fig. 6(a)) of unmitred right angle bends for W/H from about 0.25 to 2.5, as measured and computed. Points from this graph have been transferred to Fig. 6(b) of this paper, which also shows results computed by this theory. These computed results were found from the formula:

$$S_e/H = (\pi/4)W/H - 2\frac{1 - \exp(-Z_o\epsilon_e^{1/2}/60)}{Z_o\epsilon_e^{1/2}/60)} \tag{23}$$

The first term is the effective line length discussed above, and the second term employs (21) with (11) to express the incremental line lengths caused by the negative end inductances. These computations are based entirely on inductive end-effect; their agreement with the results of other investigators implies that the excess discontinuity capacitance is small.



(a)



(b)

Fig. 6. (a) Configuration and inductive circuit of right angle bend. (b) Predicted electrical length of right angle bend.

Hairpin Turns and Dogleg Bends

These configurations are shown in Fig. 7, along with their equivalent circuit. Each bend is treated as described in the above section, but it is also necessary to introduce an interaction term, L, at each end of the line joining the bends, if the line is short; that is, if L is not negligible compared to L'.

Short Circuit in CPW

A short circuit in CPW is illustrated in Fig. 8. This section will describe how the expected inductive end-effect can be evaluated. The theory of this paper cannot be applied directly to CPW, but can be applied to the complementary transmission-line [11], twin-strip, which is also illustrated in Fig. 8 by inter-changing metal and dielectric areas of the CPW.

Suppose the effective dielectric constant, ϵ_e, width ratio, a/b, and characteristic impedance, Z_{cp}, of the CPW [12] are known, and thin conductors assumed. Then, by [11], the complementary twin strip has the same ϵ_e and a/b, and its characteristic impedance, Z_{ts}, is:

$$Z_{ts} = \eta_o^2/(4\epsilon_e Z_{cp}) = 35\,481/(\epsilon_e Z_{cp}) \qquad (24)$$

Twin-strip is a two-conductor line that can be related to the two-wire line analysis of this paper by requiring the same characteristic impedance and effective dielectric constant, and specifying a valid relation between their cross-sectional dimensions. A proven formula for this relation has not been worked out, but for purposes of illustration, it will be assumed that the twin strips and the round wires have the same distance between center-lines. This should be a close approximation for high impedance twin-strip (low impedance CPW), and useful for moderate impedance levels. Employing the notation used previously for Fig. 8:

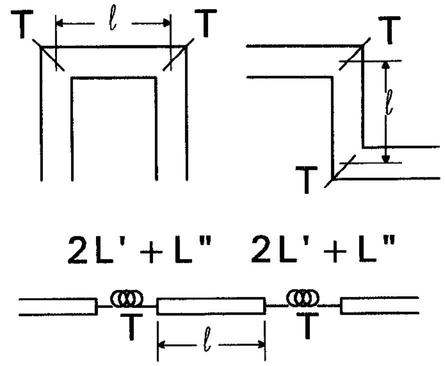


Fig. 7. Configuration and circuit for hairpin and dogleg bends.

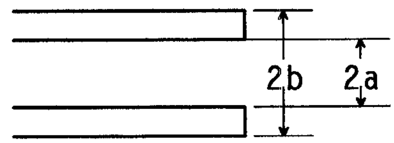


Fig. 8. Configuration of coplanar waveguide short circuit.

$$B = a + b \qquad (25)$$

With wire spacing, B, defined, wire diameter D can be found from (5) using (9), distance G from (2), and negative inductance, L', from (6). As these are balanced lines, the total end inductance is $2L'$. Being duals of the short-circuited CPW line, the twin-strip and two-wire line are open-circuited, and so their end-capacitance will be found. Equation (19) for C' will do; thus:

$$C'_{ts} = -2L'/Z_{ts}^2 \qquad (26)$$

A final transformation [11] from twin-strip back to CPW gives the end-effect inductance, L'_{cp}, of the shorted CPW line.

$$L'_{cp}(nH) = 35.481 C'_{ts}(\mathrm{pF})/\epsilon_e \qquad (27)$$

The above logical procedure can be collapsed into a single equation for the end-inductance of a CPW short circuit:

$$L'_{cp} = (2/\pi)\epsilon_o\epsilon_e(a+b)Z_{cp}^2 \cdot (1 - 1/\cosh(60\pi^2/(Z_{cp}\epsilon_e^{1/2}))) \qquad (28)$$

The end-effect inductance for a CPW short is positive, unlike that of microstrip or twin-strip.

APPENDIX

This appendix outlines the derivation of the inductance of two parallel conducting cylinders of finite length, as shown in Fig. 1(a).

It is well known [4] that the magnetic field generated by currents flowing oppositely along two parallel hollow conducting cylinders with center to center spacing B, is the same, external to the cylinders, as that generated by the same total currents on two parallel filamentary conductors spaced by distance A. A and B are related by (1). For mathematical convenience, such filamentary currents will be assumed for determining the field in the plane of the conductors. Let one filament lie along the x-axis from $-S/2$ to $S/2$, and both filaments lie in the x-y plane. The field, $dH(x, y)$, at an arbitrary point, x, y, generated by current I over a very short path, dx', located at x', is given by Ampere's law [4, p. 89]:

$$dH(x,y) = dx' I \sin(\theta)/(4\pi r^2) \qquad \text{(A1)}$$

where r is the distance between x' and x, y, and θ is the angle between the line defining r and the x-axis. In terms of x', x and y only, (A1) can be written as:

$$dH(x,y) = \frac{dx' I(y/4\pi)}{((x-x')^2 + y^2)^{3/2}} \qquad \text{(A2)}$$

The total field, $H(x, y)$, caused by the current on the filament lying along the x-axis is the integral of (A2) with respect to x' between the limits $-S/2$ and $S/2$. The result is:

$$H(x,y) = \frac{I}{4\pi y}\left[\frac{((x+S/2)}{((x+S/2)^2+y^2)^{1/2}} - \frac{(x-S/2)}{((x-S/2)^2+y^2)^{1/2}}\right] \qquad \text{(A3)}$$

An expression for the inductance, L, of a closed conducting loop [4, p. 216] calls for the surface integral of the normal magnetic flux density over the loop area. For this case, it can be written as:

$$L = (\mu/I)\int_{\text{surface}} H(x,y)\,dx\,dy \qquad \text{(A4)}$$

The limits for x are from $-S/2$ to $S/2$, and the limits for y are from $P = (A - B + D)/2$ to $Q = (A + B - D)/2$ that is, from the inner sides of the round conductors that define the loop of interest; there is no field within the conductors). Using (A3) for $H(x, y)$, the field generated by only one round conductor, results in an expression for the contribution, L, of that conductor to the total loop inductance:

$$\begin{aligned}2\pi L/\mu = {} & P - (P^2+S^2)^{1/2} \\ & + S\log((S+(P^2+S^2)^{1/2})/P) \\ & - Q + (Q^2+S^2)^{1/2} \\ & - S\log((S+(Q^2+S^2)^{1/2})/Q) \qquad \text{(A5)}\end{aligned}$$

The external inductance of a single isolated conductor of finite length, S, can be found by letting Q approach infinity in (A5); when this is done, $P = D/2$. This quantity is given as L_1 in (4). The contribution of both round conductors, denoted L_t, to the total loop inductance is twice the value of L given by (A5). Algebraic manipulation of (A5) yields the expressions of interest,

$$L_t = 2L_o + 4L' + 4L'' \qquad \text{(A6)}$$

where expressions for L_o, L', and L'' are given by (5), (6) and (7), respectively.

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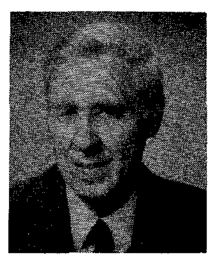

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